NEUTRALINOS AS DARK MATTER IN THE MINIMAL SUPERGRAVITY MODEL*

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Abstract

A new approach to the phenomenological study of the minimal supergravity model is presented in which the model is effectively parametrized in terms of five low energy observables. Radiative corrections due to large Yukawa couplings and particle–sparticle mass splitting are included into the analysis and found to have important effects, in particular on the degree of fine tuning in the model. In this framework the neutralino relic abundance has been calculated and the cosmologically interesting range of parameters determined (after imposing all the presently available accelerator limits).

INTRODUCTION

The topic we would like to address here is not a new one: it has already been discussed for about a decade that the lightest neutralino in supersymmetric models with R–parity conservation is one of the theoretically best motivated dark matter candidates. This idea has attracted a lot of attention at various levels of model building. It can be explored in the framework of the low energy effective Minimal Supersymmetric Standard Model (MSSM) with the most general soft supersymmetry breaking terms. It has also been discussed in the MSSM with additional "theoretically motivated" conditions, such as 1) unification constraints

$$\begin{aligned} M_1 &= \frac{5}{3}\tan^2\theta_W M_2 \approx 0.5\ M_2 \\ M_2 &= \frac{\alpha_2}{\alpha_3}\ M_3 \approx 0.3 m_{\tilde{g}} \end{aligned} \qquad (1)$$

or 2) "Supergravity Inspired Scenario" (SIS) (usually referred to as minimal supergravity model) in which all the free mass paremeters in the low energy effective lagrangian are determined in terms of 5 unification scale parameters: the superpotential Higgs mixing mass μ_o, the universal scalar and gaugino masses m_o and M_o, respectively, the universal trilinear coupling A_o and the parameter B_o.

Clearly, with more constraints the predictive power of the MSSM increases quite dramatically†. A particularly predictive framework is that of the minimal supergravity model which in addition offers the very

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†We do not discuss here at all non–minimal supersymmetric models

interesting possibility of radiatively induced $SU(2) \times U(1)$ gauge symmetry breaking. It depends on 5 parameters only (plus the yet unknown top quark Yukawa coupling Y_t) and it interconnects: 1) the mechanism of the radiatively induced $SU(2) \times U(1)$ breaking, 2) the Higgs sector, 3) the sparticle spectrum and couplings and 4) the neutralino as a dark matter candidate. Even the present experimental and cosmological limits constrain strongly so predictive model. On top of this comes the question: can this model survive the additional requirement of no fine tuning of the parameters at the unification scale M_U?

Given the interesting theoretical motivation for the SIS, it seems logical to fully explore its phenomenological predictions – and neutralino properties in particular – to eventually rule out the simplest MSSM or to collect evidence in its favour. In spite of the large number of papers devoted to the model, the situation in this respect is not yet satisfactory and only fragmentary predictions have been available.

ANALYSIS

Here we report on a recent reanalysis of the SIS–MSSM model with radiatively induced $SU(2) \times U(1)$ symmetry breaking[1]. There are three main new points in our strategy: 1) bottom–top approach, 2) inclusion of radiative corrections in all sectors, 3) consistent calculation of the neutralino relic abundance.

The bottom–top approach consists in setting up the formalism to systematically explore the full parameter space of the model by choosing a set of values for 5 *low energy* physical parameters and looking for the set of parameters at the unification scale M_U that induces the gauge symmetry breaking and gives the chosen low energy values. Thus, we reverse the standard procedure which is to find the low energy parameters corresponding to a given set of the boundary conditions at M_U. As low energy parameters we fix M_Z, M_A (the Higgs pseudoscalar mass), $\tan\beta$, $m^2_{\tilde{t}_L}$ and $m^2_{\tilde{t}_R}$ (the soft stop mass parameters; the physical stop masses are eigenvalues of the mass matrix) and we determine the corresponding m_o, M_o, μ_o, A_o, B_o at M_U by solving a set of algebraic equations, like e.g.

$$\begin{aligned} m^2_{\tilde{t}_L} &= c_1 M_o^2 + c_2 m_o^2 + c_3 \mu_o^2 \\ &+ c_4 A_o^2 + c_5 M_o A_o \end{aligned} \tag{2}$$

with the coefficients c_i obtained numerically by the RG evolution. Then, we calculate all other observables in terms of the obtained solutions for the parameters at M_U. *The model is now effectively parametrized in terms of 5 low energy observables.* Not only it is easier to implement all present (and future) experimental limits, but also one can systematically study the phenomenology of the most general MSSM compatible with SIS and the idea of radiatively induced $SU(2) \times U(1)$ breaking, with no additional assumptions about the values of the parameters at M_U (for which we have little theoretical insight).

Radiative corrections due to large Yukawa couplings and particle–sparticle mass splitting are consistently taken into account in all sectors of the model. This is achieved by using the RG equations with heavy sparticles (squarks and gluinos) decoupled at M_S which is chosen to minimize the generated by them 1–loop corrections to the mass parameters of the scalar potential

$$\begin{aligned} V &= \hat{m}_1^2|H_1|^2 + \hat{m}_2^2|H_2|^2 \\ &- \hat{m}_3^2\left(\epsilon_{ab}H_1^a H_2^b + h.c.\right) \\ &+ \frac{1}{2}\lambda_1|H_1|^4 + \frac{1}{2}\lambda_2|H_2|^4 \\ &+ \lambda_3|H_1|^2|H_2|^2 + \lambda_4|\epsilon_{ab}H_1^a H_2^b|^2. \end{aligned} \tag{3}$$

At the tree level, $\lambda_1 = \lambda_2 = \frac{1}{4}(g_1^2 + g_2^2)$, $\lambda_3 = \frac{1}{4}(g_2^2 - g_1^2)$, $\lambda_4 = -\frac{1}{2}g_2^2$. The tree level potential is to be used from M_U to M_S, with the parameters running according to the RG equations with full supersymmetric spectrum included. The parameters at M_Z are related to those at M_S by RG equations with heavy sparticles decoupled and with the tree level boundary conditions at M_S. Radiative corrections at M_Z to some of λ_i's can be very large, of order 100%. As compared to tree level analysis, to the same values of physical observables correspond now different low energy mass parameters in the potential, hence different values of the parameters at M_U and, in consequence, different correlations among physical observables. The important role of radiative corrections to the effective potential has been recognized earlier[2] but their full impact on the phenomenology of the model has become easy to explore only within our bottom–top approach.

The lightest neutralino relic abundance is calculated for each solution to our equations, with radiative corrections in all sectors being incorporated and all correlations among physical parameters taken into account. Our procedure is to obtain the relic density by integration of the annihilation cross section (times effective degrees of freedom) from the present time to the freeze–out temperature, determined by solving the appropriate freeze–out condition[4]. The lightest neutralino annihilation cross section for all channels ($\tilde{\chi}\tilde{\chi} \to f\bar{f}$, W^+W^-, ZZ, H_iH_j, H^+H^-, H_iZ, $H^{\pm}W^{\pm}$) have been recomputed in the partial–waves formalism and high energy unitarity has been used as a check. Some discrepancies with the previous literature have been found and will be detailed elsewhere[1]. Finally, threshold effects have been incorporated accoring to ref.[4].

RESULTS

One of our important findings is a strong effect of radiative corrections on the degree of fine tuning in the model. When calculating the low energy observables in terms of the high energy parameters (eq.(2) and alike) we want to avoid cancellations in the r.h.s. much larger than the final answer itself. Otherwise we have fine tuning of the parameters at M_U. It has been proposed[3] that a useful measure of fine tuning are the weighted derivatives

$$\Delta_{ij} = \frac{P_i}{Q_j}\frac{\partial Q_j}{\partial P_i}\ , \tag{4}$$

with $Q_j = \left(M_Z, \tan\beta, M_A, m_{\tilde{t}_L}, m_{\tilde{t}_R}, Y_t, Y_\tau\right)$ and $P_i = (M_o, A_o, m_o, \mu_o, B_o, Y_t^o, Y_\tau^o)$. Indeed,

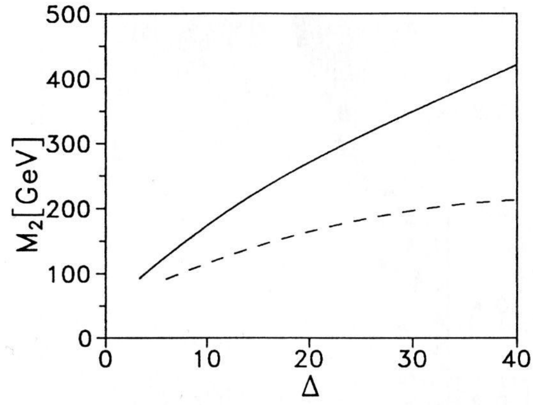

Figure 1: Maximal M_2 plotted as a function of the max$|\Delta_{ij}|$ for m_t = 160 GeV and $\tan\beta = 5$ with (solid line) and without (dashed line) radiative corrections.

max$|\Delta_{ij}|$ is directly related to the order of magnitude of the necessary cancellations. To some extend it is a matter of taste how big cancellations are still tolerable. As the coefficients c_i in equations like (2) depend on large logarithms, $\ln(M_U/M_Z) = O(30)$, we consider this to be the tolerable order of magnitude for cancellations.

Radiative corrections sizably diminish the amount of fine tuning. This is illustrated in Fig.1 where the maximal acceptable M_2 is plotted as a function of the max$|\Delta_{ij}|$ calculated for our solutions for m_t = 160 GeV and $\tan\beta = 5$. With radiative corrections, the acceptable range of M_2 is almost twice as large as without. A similar effect exists for $m_{\tilde{t}}$ and μ. The origin of this effect can be easily traced back to the behaviour of the coefficients c_i in the algebraic equations (2).

We then ask for the model predictions which are compatible with: a) naturality defined by, say max$|\Delta_{ij}| < 30$; b) present accelerator constraints which we take to be $m_{\tilde{g}} > 150$ GeV, $m_{\tilde{q}} > 100$ GeV, $m_{\tilde{l}} > 45$ GeV, $|Z_{13}^2 - Z_{14}^2| < 0.03$, $|Z_{13}Z_{23} - Z_{14}Z_{24}| < 0.04$, and similar relations for other neutralinos to which Z bosons can decay (here Z_{ij} determine the neutralino composition $\tilde{\chi}_i = Z_{i1}\tilde{B} + Z_{i2}\tilde{W}^3 + Z_{i3}\tilde{H}_1 + Z_{i4}\tilde{H}_2$); and c) the cosmological bound

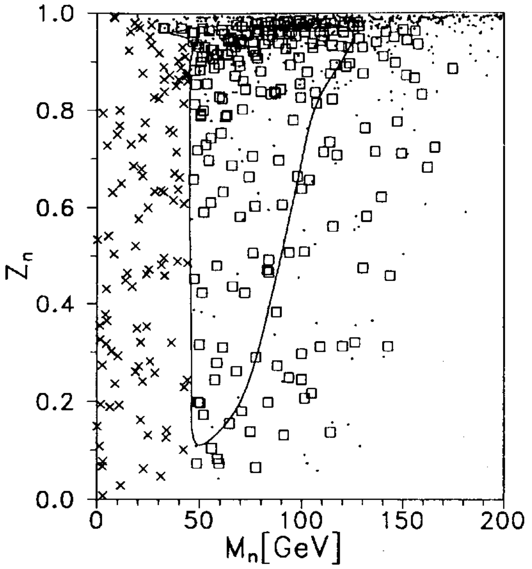

Figure 2: Gaugino content $Z_n = Z_{11}^2 + Z_{12}^2$ of the lightest neutralino plotted as a function of its mass M_n for $m_t = 120$ GeV and $\tan\beta = 5$. Crosses (squares) describe solutions with $\max|\Delta_{ij}| < 30$ which are excluded (allowed) by the present experimental data; dots correspond to solutions compatible with experimental data but with $\max|\Delta_{ij}| > 30$. With no radiative corrections included, solutions with $\max|\Delta_{ij}| < 30$ and compatible with the present experimental limits lie inside the contour line.

$\Omega h^2 < 1$ on the lightest neutralino relic density.

Our answer to these questions is illustrated in Figs. 2–10, with emphasis on the neutralino properties and on the comparison of results with and without inclusion of radiative corrections (a discussion of other predictions can be found in ref.[1]). We present our results in the form of scatter plots obtained with the input parameters M_A, $m_{\tilde{t}_L}$ and $m_{\tilde{t}_R}$ varing in steps of a few GeV's from 0 to 1 TeV.

Among the points we would like to draw attention to are:

1. Interesting correlations exist between m_t and $Z_n = Z_{11}^2 + Z_{12}^2$, the gaugino content of the lightest neutralino, between $\tan\beta$ and Z_n, and between Z_n and M_n, the neutralino mass. All of them can be

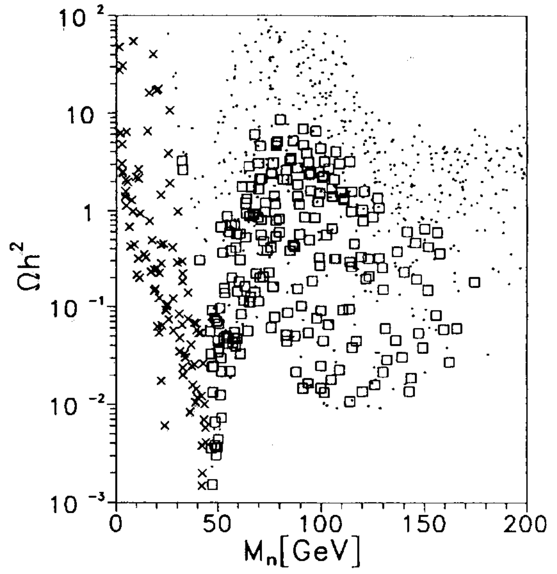

Figure 3: Relic abundance of the lightest neutralino plotted as function of its mass M_n for $m_t = 120$ GeV and $\tan\beta = 5$. Notation as in Fig.2.

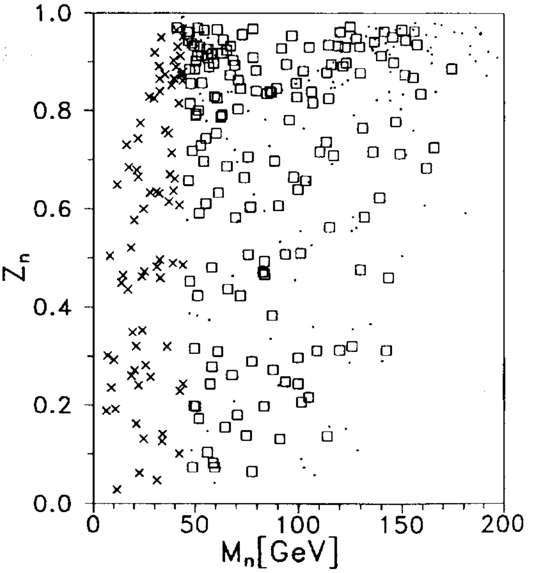

Figure 4: The gaugino content Z_n of the lightest neutralino as function of its mass M_n for solutions with $\Omega h^2 < 1$. Here $m_t = 120$ GeV, $\tan\beta = 5$.

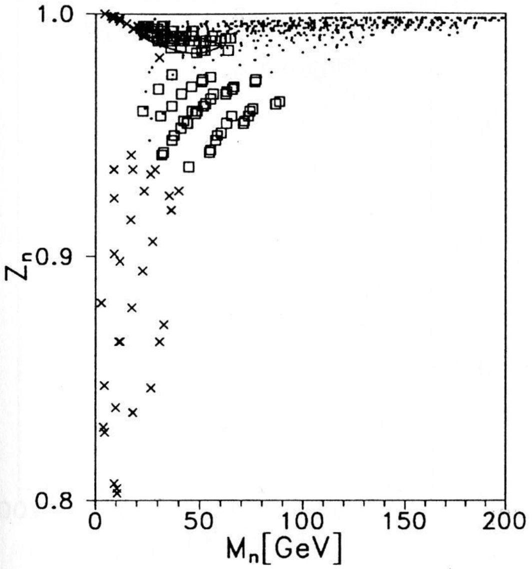

Figure 5: Same as Fig.2 but with m_t = 160 GeV, $\tan\beta = 2$.

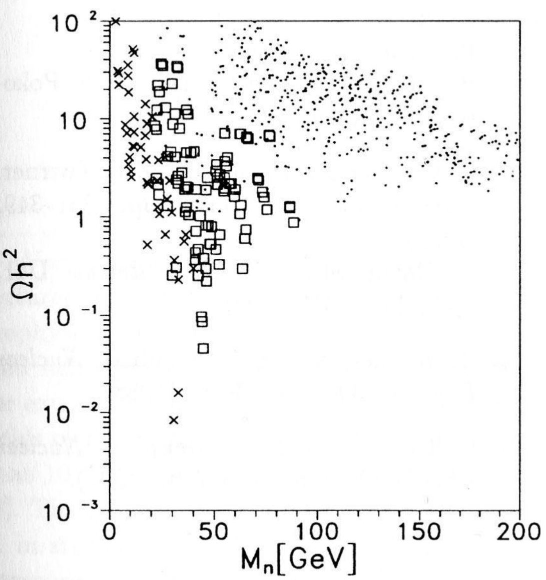

Figure 6: Same as Fig.3 but with m_t = 160 GeV, $\tan\beta = 2$.

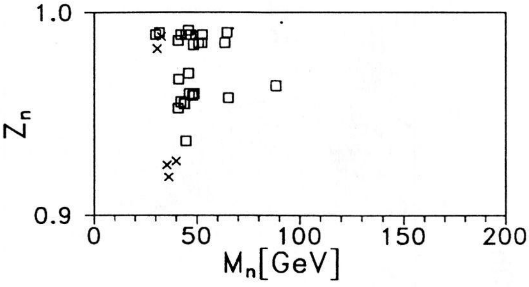

Figure 7: Same as Fig.4 but with m_t = 160 GeV, $\tan\beta = 2$.

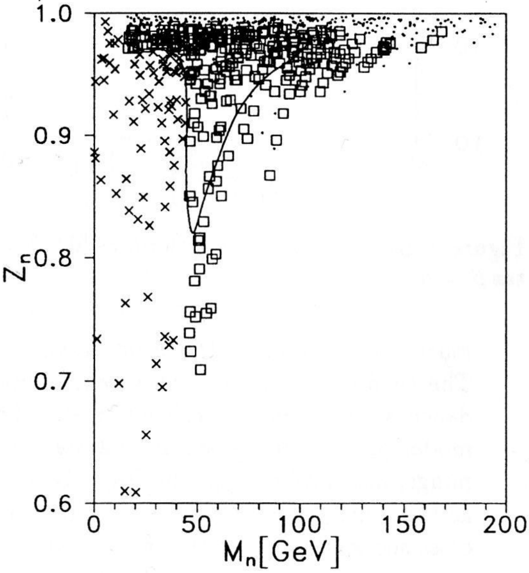

Figure 8: Same as Fig.2 but with m_t = 160 GeV, $\tan\beta = 20$.

understood qualitatively in terms of the structure of the model.

2. Results with and without radiative corrections, although qualitatively similar, differ quite dramatically at the quantitative level. In particular, without radiative corrections, the model suffers from the naturality problem and, with our criteria, is already almost excluded.

3. In a large region of the parameter space $\Omega h^2 > 1$ and hence the cosmological bound is a strong constraint. One should remember that Ωh^2 has been calculated with all the correlations between physical

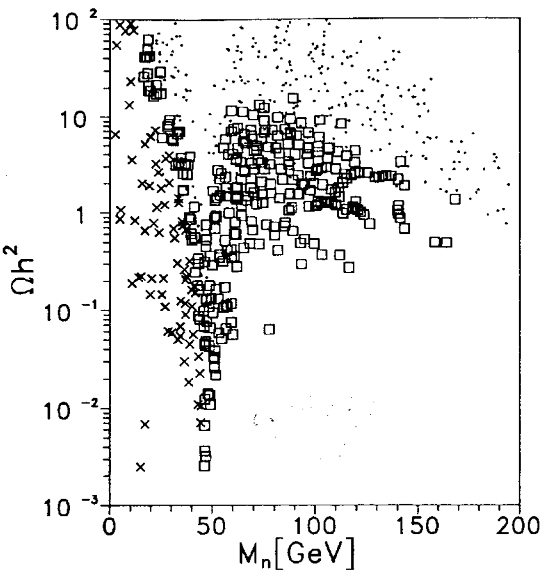

Figure 9: Same as Fig.3 but with $m_t = 160$ GeV, $\tan\beta = 20$.

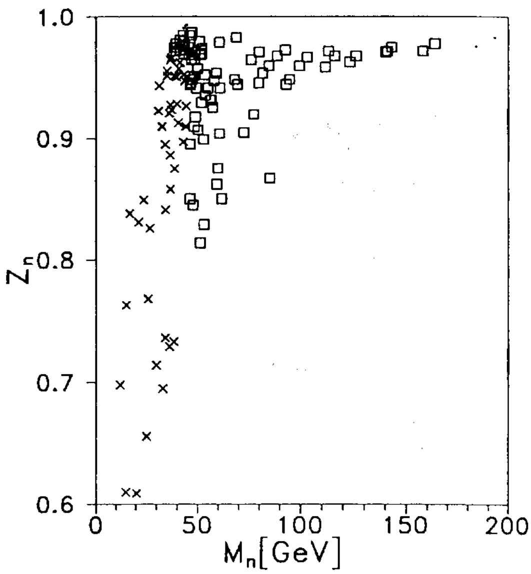

Figure 10: Same as Fig.4 but with $m_t = 160$ GeV, $\tan\beta = 20$.

masses and couplings taken into account. The tendency toward a large relic abundance is due to such generic features of the model predictions as small Yukawa couplings, heavy sfermions and higgs bosons and therefore relatively small annihilation cross sections and few channels open.

4. It is interesting to notice that the parameter regions excluded by the present accelerator data are also often excluded cosmologically. Notice also that most of the "unnatural" solutions are excluded by the cosmological bound $\Omega h^2 < 1$ (the only exception are the higgsino–like solutions for light top quark). Notice however that some of the solutions in the cosmologically interesting regions do not satisfy our no fine tuning constraint.

Our final conclusion is that, after radiative corrections included, the model remains phenomenologically very appealing, with neutralinos as a very interesting dark matter candidate.

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